Is Bigger Better? Customer Base Expansion through Word of Mouth Reputation

Arthur Fishman
Department of Economics
Bar Ilan University
Ramat Gan
Israel
afishman@biu.ac.il

Rafael Rob*
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297
rrob@econ.sas.upenn.edu

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Abstract

We develop a model in which information about a firm's past performance diffuses gradually among potential consumers. In our model, a firm's ability to deliver high-quality products at any given period depends on how much it invests in quality. This investment is the firm's private information. Also, a firm's current quality is unobservable. Thus the only observable is a firm's past performance - the realized quality of the products it delivered in the past. We assume that information about a firm's past performance diffuses only gradually in the market. Thus, the longer a firm has been delivering high-quality products, the larger the number of potential customers that are aware of it. We show that, in equilibrium, a firm's investment in quality increases over time, as its reputation - the number of consumers who are aware of its history - increases. This is because the greater its reputation, the more the firm has to lose from tarnishing it by under-investing and, conversely, the more it has to gain from maintaining it. This is recognized by rational consumers. Therefore, older - and hence larger firms - command higher prices as quality premia. This in turn feeds back into firms'

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investment incentives: the fact that they are able to command higher prices motivates older and larger firms to invest still more. So the older and larger a firm is, the more valuable an asset its reputation is.

Key words: Reputation, Adverse Selection, Investment in ability, Firm size.

JEL Classification numbers: D82, L14, L15

1. Introduction

Overview and Results. A firm's reputation is often its most valuable asset. For example, if a corporate giant like Coca Cola, McDonald's or Nike were stripped of its name - and the reputational resources associated with it - its value would be reduced to only a small fraction of what it is today. The importance of a firm's name and reputation for its balance sheet suggests that considerable managerial resources are devoted to establishing, maintaining and enhancing the value of the firm's name and reputation. The goal of this paper is to develop a modeling framework in which a firm perceives its reputation as a capital asset whose value is established, maintained and enhanced through a process of active and continuous investment.

We consider a market for a product or service whose quality is unobservable at the time of purchase. Consequently, consumers' purchasing decisions are based on what they know about a firm's past performance - the realized quality of the products it delivered in the past. Our model has two main ingredients. First, we assume that the ability to produce high-quality products requires continuous investment. Second, we assume that information about a firm's past performance diffuses only gradually in the market. This leads to a dual process of gradual quality improvement accompanied by a gradual expansion of the firm's customer base ("customer base" being the number of consumers who are aware of the firm's past performance).

Because building a reputation takes time, an older and more established firm, with a larger customer base, has more to gain from maintaining its reputation for quality and, conversely, has more to lose from tarnishing it. It therefore invests more and hence delivers higher expected quality than a younger firm. Consequently, consumers associate market tenure (firm age) with quality and are willing to pay older firms more. And the fact that an older firm commands a higher price and has a larger clientele further increases its incentive to invest.

Notwithstanding this, even established firms fail on occasion and are forced to exit. This opens the door to new entrants who start from scratch and strive to establish their own reputation. Therefore, if we consider an entire industry, which consists of many firms, and analyze its evolution through time, such industry is characterized by turnover - some firms succeed and grow, while others fail and exit. These processes of growth and decline give rise to a stable, steady state distribution over firm sizes and correspondingly a distribution over product qualities.

Empirical Support. The association between market tenure and perceived quality predicted by our model seems to fit the observation that producers of high-quality products with a long history in the market tend to emphasize this characteristic in their advertising. For example, the New York Times heralds the year in which it was founded on its front page and several European beer manufacturers vaunt the year in which the brand was established on their label. Similarly, advertising often seems to signal quality through market share. For example, the Hertz ad: "We're number one."

Further correspondence between our model's predictions and data is found in the features that firms which satisfy their customers grow, that firms that disappoint their customers decline, and that, at any given point in time, different firms are of different sizes. For example, Figure 1.1 (based on data reported by International Data Corporation; see also the case study "Matching Dell") shows the evolution of market shares of the leading PC manufacturers over the last 10 years. Figure 1.1 illustrates that, at a given point in time, firms are not equally sized and that some firms grow while others decline.

Related Literature. There is a growing literature on reputation in markets. This literature started in Klein and Leffler's (1981) paper, and extended in Shapiro (1983), Rogerson (1983), and Allen (1984). These papers consider repeated market interaction, introduce the notion that a firm's past performance is its "reputation,"

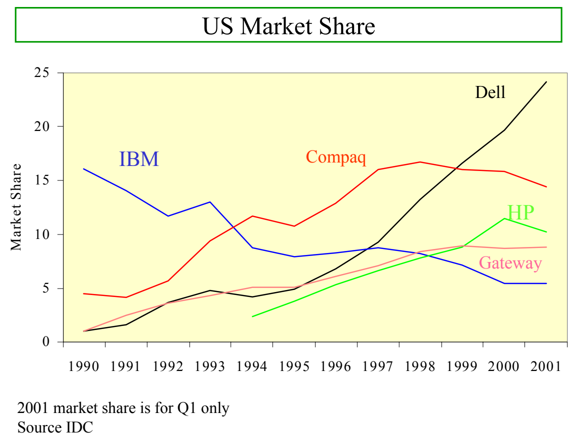

Figure 1.1: The Evolution of U.S. Market shares of PC manufacturers

and show that a moral hazard problem (quality erosion) may be solved when consumers condition their purchasing decisions on a firm's reputation. Another modeling approach, also based on repeated market interaction but introducing adverse selection (firms are of different types), is found in Diamond's (1989) credit market model, which builds on Kreps and Wilson (1982) and Milgrom and Roberts (1982). Borrowers (who have not previously defaulted) in Diamond's model have a lower posterior probability of default, and consequently pay lower interest charges, the longer their credit history. Our paper is related to these modeling approaches in that we identify a firm's reputation with its past performance, and in that we find that a firm's value increases over time as this firm establishes a reputation for itself.

Other aspects of reputation building (that we don't study here), for instance the notion that a market for reputation may develop in which names are bought and sold has been studied in papers by Tadelis (1999, 2002). Mailath and Samuelson (2001), using a Kreps-Wilson (1982) type formulation (but with different firm types), determine when high effort can be sustained as an equilibrium outcome.

Somewhat more tangential to our interest here is Watson (1999, 2002) who considers the process of reputation building in the context of a partnership game and relates it to the phenomenon of gradually increasing the stakes that partners have in the partnership. Closest to our interest is Horner (2002), who introduces competition and consumer switching and shows that competition is a 'threat' that induces firms to exert high effort.[1]

What differentiates our approach from all these papers is the way information propagates and consequently the way firms build a customer base. While in previous papers consumers are equally (albeit imperfectly) informed about firms, here consumers are differentially informed about firms. Each consumer knows only the history of the firm she is referred to and, possibly, a firm she chooses to sample. As a firm ages and assuming the firm survives, information about its history is passed on by an ever increasing number of past customers and, as a result, this firm gets more and more referrals to new customers. This process of customer accumulation determines an equilibrium in which the flows of entry and exit, the distribution over firm sizes and the distribution over product qualities are all endogenously determined. These model ingredients (word of mouth reputation and endogenous entry and exit) and predictions (link between age and investment in quality and firm-size distribution) are not found in any of the above papers.

Plan of paper. The rest of the paper is organized as follows. In the next

[1] The most important difference between Horner (2002) and us concerns the models' empirical implications, in particular the size distribution of firms and the entry of firms over time. In Horner's model firms are equally sized at any given point in time and, although the profits of surviving firms increase to infinity, there is no entry of new firms. In our model there is a distribution over firm sizes and continual entry and exit. As Figure 1.1 illustrates, the data of some industries is in greater conformity with our model than with Horner's model.

There are, naturally, differences in the way the models are set up, which give rise to these different predictions. Horner (2002) assumes away entry, whereas we assume free-entry. Also, Horner considers 2 actions (high and low effort), whereas we consider a continuum of actions. Finally, in Horner's model a firm can grow only as a consequence of the demise of other firms, whereas in our model consumers actively search for firms. So a firm can attract new customers even without the demise of other firms.

Section we set up the model, and in Section 3 we analyze it. Section 4 discusses the equilibria we derive, and Section 5 discusses our underlying assumptions.

2. The Model

Time is discrete and the horizon is infinite. There is a continuum of firms and consumers. The measure of consumers is one, while the measure of firms is endogenous, and yet to be determined.

There are two product-quality levels, high and low. A consumer's utility from one unit of the high-quality product is 1, and her utility from one unit of the low-quality product is 0. Each consumer lives one period and demands either one or zero units.

Firms are differentiated with respect to their ability to produce a high-quality product. A low-ability firm only produces a low-quality product, while a high-ability firm only produces a high-quality product.[2] A low-ability firm can not become high-ability. But, a high-ability firm can deteriorate and become low-ability. For example, the firm may lose key management or key employees, or allow its 'corporate culture' to deteriorate. Once a firm becomes low-ability, it can not become high-ability again. A high-ability firm can reduce the probability of becoming low-ability by investing in ability. Specifically, at the beginning of each period, a high-ability firm invests $x \in [0, \overline{x}]$, where $\overline{x} < \infty$. It then remains high-ability and produces high-quality products that period with probability $f(x)$; it becomes a low-ability firm and produces low-quality products with probability $1 - f(x)$. $f(\cdot)$ is strictly concave, strictly increasing and takes values in $[0, 1)$. In particular, $f(\overline{x}) < 1$; thus, although a high-ability firm can reduce the likelihood of becoming low-ability, it cannot eliminate this possibility altogether.[3] Apart

[2] Thus the informational issue is adverse selection rather than moral hazard.

[3] This specification assumes that the effectiveness of investment in remaining high-ability is independent of the length of time, τ, during which the firm has been high-ability. If f increases in τ, which is more natural in some applications, the result we derive below still holds. We

from x, which is independent of the scale of the firm's output, the firm's variable cost of production is zero (it can be any constant < 1). A firm's production level is not subject to capacity limits.

The same is true for a new entrant; if it invests x, it is "born" as a high-ability firm with probability $f(x)$ and as a low-ability firm with probability $1 - f(x)$. To be operative at any period, a firm must pay a non-recoverable fixed cost of $F > 0$ at the beginning of the period. This cost can be saved by exiting.

A firm's investment, x, is its own private information. The quality which is about to be realized as a result of investing x is known neither to the firm nor to consumers. However, once the product is purchased (if at all) and used, its realized quality, which is the same for all units sold, becomes known to the firm and to consumers who bought this product, also referred to as the firm's customers. Thus, last period's quality is observed by the firm and its customers. Last period's product-quality can also be ascertained by consumers who find the firm by searching, where the meaning of "searching" is explained below.

Because a firm's *current* quality and its investment are unobservable, we define a **firm's type** by the pair (t, q), where:

(i) q is its realized product-quality at the *preceding* period. $q = low$ if the firm delivered a low-quality product last period, in which case the firm is known (to itself, last period's customers and consumers who search the firm) to be a low-ability firm from this point onwards. $q = high$ if the firm delivered a high-quality product last period.

And,

(ii) t is the firm's age - the time elapsed since the firm entered the market.

The reason age is relevant is that, as shall be seen below, it determines the firm's customer base - the number of customers to which the firm has access. And that variable determines the firm's investment level, and hence its expected

discuss this possibility in Section 5.

quality.

At each period a new generation of risk-neutral consumers of measure 1 enters the market. Each consumer lives one period. When they enter the market, consumers know only the distribution of firm types but not which firm is what type. Upon entering the market, a new consumer meets one old consumer of the previous generation who reveals to her the type of the firm from which she bought at the preceding period.[4] We call this firm the new consumer's **reputation firm**, and if the consumer buys from this firm we call her a **reputation customer**.

A consumer can either buy from her reputation firm, or she can search a randomly selected firm.[5] In the latter case the consumer is called a **search customer**. We assume that when a consumer finds a firm by searching, she learns the firm's type. Thus, whether the consumer is a reputation customer or a search customer she knows the firm's type and can base its purchase decision on this information. For simplicity, we assume that, because of high search costs or time constraints, a consumer can only search once. Also, if a consumer searches, she must either buy from its search firm or leave the market without buying - she can not go back to her reputation firm after searching. In this environment consumers search if the average surplus from searching exceeds the surplus they get from their reputation firm. We assume searching consumers are divided uniformly across firms, i.e., each firm receives the same number of search customers.

When a consumer is matched with a firm, the two are in a short-term bilateral monopoly situation. It is natural in this situation to assume that prices are determined through bargaining. Accordingly, if the consumer and the firm believe the expected quality of the firm's product to be Q (in equilibrium firms and consumers have common-knowledge beliefs), the consumer pays γQ and gets a

[4] The qualitative features of the equilibria we describe below remain intact if an entering consumer is referred to a firm with probability less than one but greater than zero or if she is referred to more than one firm.

[5] If a consumer is referred to an exiting firm (which, we show, happens in equilibrium) the consumer must search.

consumer surplus of $(1-\gamma)Q$, where γ is an exogenously fixed parameter $\in (0,1)$. γ represents the firm's bargaining skill; the bigger is γ, the larger is the fraction of the trading surplus that the firm captures.[6] We explain in Section 5 what happens if γ is either 0 or 1.

Recall that consumers only observe a firm's type, not its actual investment. We denote consumers' belief (also referred to as consumers' expectation) about a firm's investment level, as a function of its type, as $b(t,q)$. This fully determines consumers' belief about the expected quality of this firm's product, $f(b(t,q))$.

A consumer decides whether to accept her reputation firm, i.e., buy from this firm and not search. We let A be the consumer's acceptance set, i.e., $(t,q) \in A$ if and only if the consumer buys from her reputation firm of type (t,q), and searches otherwise.[7]

A firm of type (t,q) makes two decisions. (i) It decides whether to remain operative by paying F or exit. If a firm exits its outside option is worth zero. And, (ii) if the firm remains operative, it decides how much to invest, $x(t,q)$, in remaining high-ability. We refer to the dependence of exit and investment on type as the firm's **decision rule**.

Let us summarize this description by stating the timing convention within a single period. At the beginning of a period a firm that was operative last period decides whether to remain operative for another period and if so it chooses its investment x in remaining a high-ability firm. Only firms that were revealed to be high-ability in the previous period choose x; low-ability firms are unable to affect their ability. At the same time new firms enter and choose their x. Next a new generation of consumers arrives, and consumers are matched to firms, based on

[6] A concrete interpretation of γ is that, upon meeting, the firm or the consumer is randomly chosen to make a take-it-or leave-it offer to the other, and that γ is the probability with which the firm is chosen to make the offer. Under this interpretation γQ is the ex-ante (i.e., before the identity of the proposer is known) expected surplus of the firm. γ is a measure of bargaining skill in this situation because whoever makes the offer extracts the full surplus, and γ measures how likely the firm is to seize this opportunity.

[7] If a consumer is indifferent she does not search.

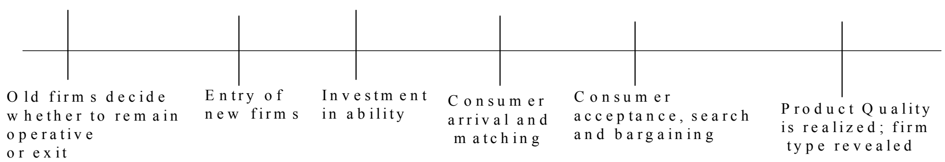

Figure 2.1: Timing convention within a single period

the information they receive. Then each consumer decides whether the firm she is matched to is acceptable and if so she bargains with this firm over the product price; if the firm is not acceptable the consumer searches a new firm and bargains with it over the product price. Finally, the realized quality of each firm's product and hence the current period ability of this firm are revealed. This brings the period to an end. This timing convention is summarized in figure 2.1.

The number and composition of firms in the market is determined by the equilibrium flows of entry and exit. The flow of entry is such that the discounted value (net of the initial F) of new firms is zero.

We seek a **rational-expectations steady-state equilibrium**. Such an equilibrium is characterized by:

- The measure of new entrants per period, e.
- An investment rule - $x(t, q)$ - and an exit rule for a firm of each type.
- An acceptance set, A, for consumers and consumers' investment expectations, $b(t, q)$.

Such that:

1. Firms' decisions maximize future discounted profits, and consumers' decisions maximize utility.
2. Consumers' expectations are correct; $b(t, q) = x(t, q)$.
3. Entrants earn zero discounted profit, ex ante.
4. There is a constant measure of firms of each type.

3. Analysis

We first prove that in *any* equilibrium, young firms start out by making negative period profits. Then as firms age they charge higher prices, invest more in remaining high-ability, deliver, on average, higher-quality products, have more customers, enjoy higher period profits and enjoy higher stock market valuations. After we establish that, we prove the non-vacuousness of these properties, i.e., we prove the existence of an equilibrium. We start out with the following result.

Lemma 3.1. *In any equilibrium, (t, low)-firms exit at once for any t.*

Proof. Since all potential buyers know a firm's type before buying from it, they pay zero for the product of a low-ability firm. Hence low-ability firms earn a period profit of $-F$ while in the market and should optimally exit. ■

Since, by the preceding Lemma, only $(t, high)$-firms are operative, a firm's type will be denoted, from this point on, by its age, t. Correspondingly we refer to type-t firms instead of type-$(t, high)$ firms. Let the measure of type-t firms be n_t and let their investment and expected investment be x_t and b_t. Let $n = \sum_{t=1}^{\infty} n_t$ be the measure of all firms (firms of all types).

Let ν be the measure of firms of type $t \in A$. Then there are ν reputation consumers and $1 - \nu$ searching consumers. Since searching consumers are divided uniformly across firms, each firm (be it a firm in A or not in A) receives $y = (1-\nu)/n$ search consumers.

In equilibrium firms know b_t and, hence, the expected utility to a consumer from a unit sold by a firm of type t, $f(b_t)$, is common knowledge between the firm and its potential customers. Thus the price a type-t firm receives for each unit it sells is $p_t = \gamma f(b_t)$. This price is independent of the firm's actual investment x_t; it depends solely on what consumers' *believe* its investment to be, b_t.

A firm does not have access to all consumers. It can only sell to consumers who learn about it either by reputation or by search. We denote by z_t the number of customers to which a type-t firm has access and refer to this variable as the firm's **customer base**. Note that for $t \in A$, $z_t = z_{t-1} + y$, and for $t \notin A$, $z_t = y$.

Let R_t be the maximized value of a firm of type-t. Each of its customers pays p_t, giving a period profit of $-F - x_t + z_t p_t$. With probability $f(x_t)$ the firm produces high-quality products this period and hence remains operative next period. And with the complementary probability $1 - f(x_t)$ it produces low-quality products, exits, and earns zero in the future. Hence, R_t satisfies the following recursion:

$$R_t = \max\{0, -F + z_t p_t + max_x[-x + \delta f(x) R_{t+1}]\}, \tag{3.1}$$

where $\delta \in (0,1)$ is the discount factor. By the usual dynamic programming arguments, see Stokey, Lucas and Prescott (1989), (3.1) has a unique solution. And, because f is strictly concave, the sequence of maximizers, $(x_t)_{t=1}^{\infty}$, is unique, too.

The following properties of (3.1) are used in the sequel. The proof, which is straightforward, is omitted.

Lemma 3.2. *(i) The maximizer on the RHS of (3.1) is strictly increasing in R_{t+1} and independent of z_t and p_t. Conversely, if $x_t > x_{t'}$ then $R_{t+1} > R_{t'+1}$. (ii) A firm invests zero, $x_{t-1} = 0$, if its future discounted profit, R_t, is zero. (iii) If $R_{t+1} \geq R_t$ and $z_t p_t \geq z_{t-1} p_{t-1}$, with at least one of these inequalities holding strictly, then $R_t > R_{t-1}$.*

The above Lemma suggests that age, size and investment in quality are "complements," i.e., they grow together over time. To illustrate this idea, assume that as a firm ages, its value, R, goes up. Then, according to the above Lemma, older firms invest more in quality and, consequently, enjoy a higher product-price. Furthermore, older firms deliver a higher surplus to their customers, and, consequently, consumers that are referred to such firms are likely to accept these firms, rather than search. Thus the volume of sales of older firms is bigger. But this implies that the value of older firms is higher and the cycle starts anew.[8]

All this, however, is predicated on the assumption that R increases in t in the first place. To complete the argument, this assumption needs to be validated. This is the goal of the next Proposition, which is the main result of our paper.

A sequence x_t is said to be **strictly increasing** if $x_t > x_{t-1}$ for all $t \geq 2$; it is said to be **weakly increasing** if $x_t \geq x_{t-1}$ for all $t \geq 2$, with at least one inequality being strict. Analogous definitions apply when "increasing" is replaced by "decreasing."

Proposition 3.3. *In any steady-state equilibrium investment and, hence, maximized values are strictly increasing with age.*

Proof. The proof is executed in four steps.

Step 1: x_t is not a constant.

Proof of step 1: Suppose x_t is a constant, say $\widehat{x}$. Then, in equilibrium, $b_t = \widehat{x}$. Thus each firm offers the same surplus, reputation customers do not search and each firm gets the same price, $p(\widehat{x})$. Thus, $t \in A$ for all t, which implies $z_t > z_{t-1}$. Thus, since the customer base of each firm keeps increasing and since the price it gets is constant, the gross profit, $z_t p_t$, keeps increasing. This implies

[8] Technically this Lemma allows us to make backward inferences. The larger the future value of a firm is, the larger its current investment, the higher is the price and the higher is its present value.

the maximized value, R_t, also keeps increasing: $R_{t+1} > R_t$. But this, by the foregoing Lemma, implies $x_t > x_{t-1}$, a contradiction.

Step 2: x_t cannot be weakly decreasing.

Proof of step 2: Suppose x_t is weakly decreasing. Then, for large enough t, x_t is below the average x and, a fortiori, below x_1. Therefore, there must be a $\bar{t} > 1$ such that, for $t \geq \bar{t}$, $t \notin A$. But then firms of age $t \geq \bar{t}$ have only search customers. That is, for $t \geq \bar{t}$, $z_t = y$. Moreover, since by assumption, $b_t = x_t$ is weakly decreasing, p_t is weakly decreasing as well. Hence, since both the customer base and the price are weakly decreasing we must have, for all $t \geq \bar{t}$, $\widehat{R}_t < R_1 = 0$, where $\widehat{R}_t$ is the right-hand term in the braces of (3.1) and $R_1 = 0$ derives from the free-entry condition. But this implies that firms of age $t \geq \bar{t}$ optimally exit. Consider now a firm of age $\bar{t} - 1$, which is destined to exit the following period. Then, this firm invests zero at $\bar{t} - 1$. And, in a rational expectations equilibrium, consumers know this so the price this firm gets is zero. Therefore firms of age $\bar{t}-1$ exit at once and $R_{\bar{t}-1} = 0$. And so on. Thus all firms must exit, which cannot be the case in a steady-state equilibrium. Thus x_t cannot be weakly decreasing.

Step 3: x_t is weakly increasing.

Proof of Step 3: By steps 1 and 2, there must be a minimal t so that $x_{t+1} > x_t$ and $x_{t-1} \geq x_t$. But then, by Lemma 3.2, $R_{t+2} > R_{t+1}$ and, since consumers expectations are correct, $p_{t+1} > p_t$ and $z_{t+1} \geq z_t$ (a type-$t+1$ firm offers a higher surplus than a type-t firm and, thus, if consumers accept a firm of type-t they also accept a firm of type-$t+1$, which implies $z_{t+1} \geq z_t$). Thus, we have $p_{t+1}z_{t+1} > p_t z_t$, which together with $R_{t+2} > R_{t+1}$ implies $R_{t+1} > R_t$. But this, together with Lemma 3.2, implies $x_t > x_{t-1}$. If $t > 1$ this contradicts the minimality of t. Hence, we must have $t = 1$, i.e., $x_2 > x_1$. Assume now x_t is not weakly increasing. Then, there must be a $t > 1$ for which $x_{t+1} < x_t$. But then by the exact same arguments as above (except that all inequalities are reversed), $x_t < x_{t-1}$. And, repeating this argument, we conclude $x_2 < x_1$, which is a contradiction. Hence, x_t must be weakly increasing.

Step 4: x_t is strictly increasing.

Proof of Step 4: Suppose not. Then there exists a t' such that either (i) $x_{t'+1} > x_{t'}$ and $x_{t'} = x_{t'-1}$ or (ii) $x_{t'+1} = x_{t'}$ and $x_{t'} > x_{t'-1}$. Consider (i). In that case, by the exact same argument as in Step 3, $R_{t'+1} > R_{t'}$, which implies $x_{t'} > x_{t'-1}$, a contradiction. Consider (ii). Then either t' and $t'+1 \in A$ or t' and $t'+1 \notin A$. In the first case, for all $t > t'$, $z_t > z_{t-1}$, which implies $R_{t'+2} > R_{t'+1}$ which implies $x_{t'+1} > x_{t'}$, a contradiction. In the second case, there exists a $t'' > t'+1$ such that $z_t = z_{t'}$ for $t'' > t \geq t'$ and $z_{t+1} > z_t > z_{t'}$ for $t \geq t''$, i.e., t'' is the smallest $t \in A$. By discounting, this implies that $R_{t'+2} > R_{t'+1}$, which implies $x_{t'+1} > x_{t'}$, a contradiction. This completes the proof. ■

Thus, in any steady-state equilibrium, investment and continuation profits increase with age, which implies that a firm exits only if it becomes low-ability. We turn now to the proof that such an equilibrium exists. To that end, it is useful to introduce the following notation and concepts.

Let us fix the flow of search customers per firm, y, and a sequence of consumers' expectations, $b = (b_t)_{t=1}^{\infty}$, with $b_t \in [0, \overline{x}]$ and with $b_{t+1} > b_t$ for $t = 1, 2, ...$ Since consumers expect older firms to invest more, the surplus associated with buying from older firms is bigger (this is no longer true if $\gamma = 1$, i.e., if firms capture the whole surplus). It follows, then, that consumers accept a type-t firm if and only if t is large enough, i.e., if and only if $t > T(b)$ for some "cutoff" age $T(b)$. $T(b)$ is determined as follows.

b induces a distribution over firm types, call it $\mu(b) = (\mu_t(b))_{t=1}^{\infty}$. The measure of 1-year old firms under this distribution is proportional to 1 and the measure of t-year old firms is proportional to $\prod_{\tau=1}^{t-1} f(b_\tau)$, i.e., $\mu_1(b) = \frac{1}{1+\sum_{s=2}^{\infty}\prod_{\tau=1}^{s-1} f(b_\tau)}$ and $\mu_t(b) = \frac{\prod_{\tau=1}^{t-1} f(b_\tau).}{1+\sum_{s=2}^{\infty}\prod_{\tau=1}^{s-1} f(b_\tau)}$ for $t = 2, 3, ...$ The average surplus that a consumer is looking at if she is to search once under $\mu(b)$ is proportional to $s(b) = \sum_{t=1}^{\infty} \mu_t(b) f(b_t)$. Since $s(b)$ is the average of an increasing sequence, $f(b_\tau)$, there must be an integer $T(b)$ so that $f(b_{T(b)}) < s(b) \leq f(b_{T(b)+1})$. This $T(b)$ is the cutoff age which characterizes consumers' optimal search rule.

Given y and b, the firm's objective is written as follows:

$$\underset{(x_t)_{t=1}^{\infty}}{Max} \left\{ \Pi(x_1, x_2, ... \mid y, b) \right\}, \tag{3.2}$$

where

$$\Pi(x_1, x_2, ... \mid y, b) \equiv \sum_{t=1}^{\infty} \delta^{t-1} \prod_{\tau=1}^{t-1} f(x_\tau)[-F - x_t + z_t p_t], \tag{3.3}$$

$p_t = \gamma f(b_t)$, $z_t = y$, for $t = 1, 2, .., T(b)$ and $z_t = y[t - T(b)]$ for $t > T(b)$. (3.2) is an alternative way of expressing the recursion, (3.1). Since the maximum to (3.1) is unique, for any (y, b), Π is uniquely maximized by some $x = (x_t)_{t=1}^{\infty}$. Denote this maximizer by $x = g(y, b)$. Or, when y is fixed, $x = g(b)$.

We prove now the existence of an equilibrium in two steps. In the first step, Lemma 3.4, we fix the flow of search customers that each firm gets, y, and prove that consumers' expectations, b, exist, which constitute a fixed point of g: $b = g(b)$. Under these expectations, the investment level which maximizes firms' profits, $g(b)$, coincides with b. This shows that the equilibrium requirements that consumers and firms maximize and that consumers' expectations are correct can be made consistent, i.e., requirements 1 and 2 in the definition of equilibrium are satisfied. This leaves us with the task of satisfying the zero-profit requirement and finding a steady-state distribution over firm types, requirements 3 and 4. This is done in the second step, Lemma 3.5.

Lemma 3.4. *Fix y. Then, there exists a b so that the solution to (3.2) satisfies $b = g(b)$.*

Proof. We endow $X \equiv [0, \overline{x}]^{\infty}$ with the topology of weak convergence, which turns it into a convex, compact, linear topological space. Thus, if we show g is continuous, it has a fixed point and we are done (Glicksberg, 1952). Let $(b^n)_{n=1}^{\infty}$ be a convergent sequence in X, and let $x^n = g(b^n)$. Then:

$$\Pi(x^n \mid y, b^n) \geq \Pi(x \mid y, b^n), \text{ for all } x \in X. \tag{3.4}$$

Let x^∞ be a limit point of x^n. Then, when we pass to the limit on both sides of (3.4), we get:

$$\Pi(x^\infty \mid y, b) \geq \Pi(x \mid y, b), \text{ for all } x \in X. \tag{3.5}$$

Consequently, x^∞ is a maximizer of $\Pi(\cdot \mid y, b)$. But, since Π is maximized uniquely, $x^\infty = g(b)$, i.e., g is continuous. ■

Lemma 3.4 guarantees, for every y, the existence of an b which is a fixed point of g under y, $b = g(y, b)$. Call this fixed point $\beta(y)$. Substitute $\beta(y)$ into the profit function Π and call the resulting function $\pi(y)$:

$$\pi(y) \equiv \Pi(\beta(y) \mid y, \beta(y)).$$

$\pi(y)$ is the profit (gross of the initial fixed cost F) of a new entrant when each firm gets y search customers per period and firms invest $\beta(y)$. $\pi(y)$ is increasing and continuous in y, goes to zero as y goes to zero, and goes to infinity as y goes to infinity. Thus, there exists a y^* so that $\pi(y^*) = F$. By construction, if each firm gets y^* search customers per period, then new entrants make zero profits. It remains, then, to show that a suitable choice of the entry flow, e, guarantees that each firm gets exactly y^* search customers in each period.

Lemma 3.5. *There exists an entry flow, e^*, so that each firm gets a flow of y^* search customers. e^* along with $\beta(y^*)$ induce an equilibrium.*

Proof. Assume e new firms enter each period and assume that type-t firms invest $x_t = \beta_t(y^*)$. Let n_t be the steady-state measure of firms of age t under these assumptions. That is, $n_1 = e$, $n_2 = f(x_1)e$, $n_3 = f(x_2)f(x_1)e$ and so on. Let $n = (n_1 + n_2 + ...)e$ be the steady-state measure of all firms in the market.

By the definition of n_t and the fact that $f(\cdot)$ is bounded away from 1, $n_1+n_2+....$ converges so n is finite.

Assume we want each firm to receive a flow of y^* search customers. Let ν be the measure of acceptable firms under $\beta(y^*)$. Given that search consumers divide equally between all firms, the flow of search customers that each firm gets is $y = (1-\nu)/n = (1-\nu)/(n_1 + n_2 + ...)e$. Thus, there exists a unique e^* which induces y^*, namely, $e^* = (1-\nu)/(n_1 + n_2 + ...)y^*$. By construction, e^* along with $\beta(y^*)$ constitute an equilibrium. ■

Taken together the last two Lemmas imply:

Proposition 3.6. *There exists an equilibrium in which investment, price, firm size, and firm value increase with age.*

4. Discussion of the Equilibrium

The equilibrium described by the preceding proposition divides a firm's life cycle into two phases. In the first phase, firms sell only to randomly arriving search customers. At this stage firms invest in building their reputation but do not yet experience an increase in their customer base. Nonetheless, even at that stage, older firms command higher stock market valuation and invest more in quality. This is because the time at which their customer base will begin to increase and the returns from reputation will begin to be realized are nearer at hand. Firms begin to attract reputation customers (and thus build up a customer base) only once and if they survive beyond this initial stage.

Similar features arise during the second phase, except that they are reinforced by the build up of a customer base. Older firms invest more than younger ones because the value of reputation increases with age. The longer its tenure in the market, the greater the number of potential customers who are aware of a firm's quality history. And since it takes time to build a customer base, older firms - with a greater vested interest in this valuable asset - invest more to maintain it than younger ones, whose customer base is as yet non-existent. Consumers, in turn, rationally anticipate this and hence are willing to pay more to older firms, as quality premia. This further increases the returns from reputation and the incentive to invest in it. The net result is that an age/reputation effect is operative during both phases, i.e., throughout a firm's life cycle.

5. Discussion of the Assumptions

It is useful to review the role of our assumptions for the results developed above. Our main assumptions are that (i): Consumers are imperfectly informed about firms' types (a consumer knows only the type of its reputation firm and, if he searches, the type of one other randomly selected firm) and (ii): Firms' current expected quality depends on private investment.

Suppose that only (ii) holds; i.e., consumers are perfectly and costlessly aware of the type of each firm. In that case, each firm would effectively be a reputation firm of each consumer and so age would not matter. That is, then young and old firms would face identical incentives to invest, hence would invest identically, provide identical expected quality and command the same price. Alternatively, suppose (i) holds but a firm's ability is independent of its investment. In that case, firms' expected quality would be independent of how many consumers know its type. So, again, consumers' willingness to pay would be independent of age. Thus, both assumptions are needed to drive the results of the model.

One other of our assumptions bears comment. We assume that a high-ability firm always produces high quality and a low-ability firm always produces low quality. This is somewhat extreme. Realistically, even a high-ability firm might sometimes fail to produce high quality, despite its best efforts. And similarly, even a low-ability firm might sometimes get lucky and provide high quality (Rogerson's model (1983) is in this spirit). For example, not even the best physician cures all his patients and some patients of even a poor physician get better. Similarly, even a bad lawyer wins some cases and even the best one loses some cases. A more realistic formulation might therefore be a probabilistic one: A high-ability firm produces high quality with a higher probability than a low ability one but may on occasion "fail" and produce low quality. Under those conditions, rational (Bayesian) consumers would not infer that a firm is low-ability on the basis of a single failure, but would rather assess a firm's ability on the basis of the entire history of its past successes and failures. We argue that our main result - that older firms command higher prices and invest more - will also obtain under this formulation, but now there are two forces at work. Under the probabilistic formulation, it is easier for consumers to distinguish between a high- and low-ability firm the older it is - because there are a greater number of observations on an older firm. Thus, on average, old high-ability firms would command higher prices than young high-ability firms even if investment were independent of age. But this increases the incentive of older high-ability firms to invest. Hence, under the probabilistic formulation, we expect that older high-ability firms will on average command higher prices, and hence invest more, than young high-ability firms, not only because they have accumulated more reputation customers but also because age provides more information about inherent ability.

Let us comment now on the assumption that the effect of investment on ability is constant, i.e., that the probability of remaining high ability, $f(x)$, depends only on the amount invested and is independent of the firm's age. In some contexts of interest a better assumption is that, because of learning by doing, the same investment is more effective the longer the firm has been operative. That is, the $f(\cdot)$ function shifts up after each success. Under such formulation, our result that older firms deliver higher expected quality and command higher prices would only be strengthened. The only difference would be that under those circumstances, it would not necessarily be the case that older firms invest more. If learning from experience is of sufficient importance, older firms might deliver higher expected quality without investing more than newer ones or even by investing less. But in either case older firms deliver higher quality.

Finally, let us comment on the assumption that $\gamma \in (0, 1)$, i.e., that when a consumer is matched to a firm, both get a positive fraction of the surplus. If firms were to capture the whole surplus, $\gamma = 1$, consumers would have no incentive to buy from the firm they are referred to because all the surplus from investment in quality would accrue to the firm. But then there would be no build-up of a customer base, which implies that firms would invest a constant amount in quality (that is independent of age) and the product price would also be a constant. If, at the other extreme $\gamma = 0$, firms stand to make negative profit (taking into account the fixed cost F) so they would never enter and the product would never be supplied. At both these extremes ($\gamma = 0$ or 1), the dynamic features of the reputation mechanism we described degenerate. Thus, it is important that the division of surplus is not trivial.

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